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Domatic Subdivision Stable and Just Excellent Graphs

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Abstract - A graph G is said to be domatic subdivision stable(dss), if d (G) = d (G_{sd}uv), for all u, v $\in$V (G), u $\perp$ v. A graph G is said to be Just excellent (JE), if it to each u $\in$ V, there is a unique γ - set of G containing u. Adding an edge to a dss graph results in a graph which may or may not be dss. In this paper, we have obtained the domatic partition of a JE graph upon edge addition between any pair of vertices u, v $\in$ G, u not adjacent to v and we have proved that JE graph is not a dss graph.

Keywords: *domatic; subdivision; just excellent graph; subdivision stable graph.*

I. INTRODUCTION

We consider only simple connected undirected graphs G = (V, E). Edge addition is a local operation on a graph. If u and v are nonadjacent vertices of G, then G + e. where e = (u v) denotes the graph obtained from G by adding edge e. P_n denotes the path of length n. The open neighborhood of vertex v $\in$ V (G) is defined by N (v) = { u $\in$ V (G) | (u v) $\in$ E (G) } while its closed neighborhood is the set N [v] = N (v) $\cup$ { v }. We indicate that u is adjacent to v by writing u $\perp$ v. For details on graph theory we refer to [1].

A set of vertices D in a graph G = (V, E) is a dominating set if every vertex of V – D is adjacent to some vertex of D. If D has the smallest possible cardinality of any dominating set of G, then D is called a minimum dominating set — abbreviated MDS. The cardinality of any MDS for G is called the domination number of G and it is denoted by γ(G). A γ - set denotes a dominating set for G with minimum cardinality.

The private neighborhood of v $\in$ D is defined by pn[v, D] = N (v) – N (D – { v }). A vertex v is said to be a, down vertex if γ (G – u) < γ (G). A vertex in V – D is k – dominated if it is dominated by at least k – vertices in D.

The concept of the domatic number was introduced by Cockayne and Hedetniemi in 1977. A domatic partition of a graph G = (V, E) is a partition of V into disjoint sets V_1, V_2, ..., V_K such that each V_i is a dominating set for G. The domatic number is the maximum number of such disjoint sets and it is denoted by d (G).

D. F. Rall has proved that a domatically critical graph G is domatically full if d (G) $\leq$ 3. Also he provided some examples to prove that this result does not extend to the case d (G) > 3 [2].

A subdivision of a graph G is a graph resulting from the subdivision of edges in G. The subdivision of some edge e with endpoints { u, v } yields a graph containing one new vertex w, and with an edge set replacing e by two new edges, { u, w } and { w, v }.We shall denote the graph obtained by subdividing any edge (u v) of a graph G, by $G_{sd}uv$. Let w be a vertex introduced by subdividing uv. We shall denote this by $G_{sd}uv =$ w. For details on domination we refer to [3].

A vertex v is said to be good if there is a γ - set of G containing v. A graph G is said to be excellent if every vertex of G is good. Michael A. Henning has provided a constructive characterization of total domination excellent trees in [4]. In [5], Tamara Burton et. al have determined the relation between γ - excellent, critically dominated, end dominated and dot – critical trees are equivalent

In [6] and [7] M. Yamuna and N. Sridharan, had defined a graph G to be Just excellent (JE), if it to each u $\in$ V, there is a unique γ - set of G containing u.

Example:

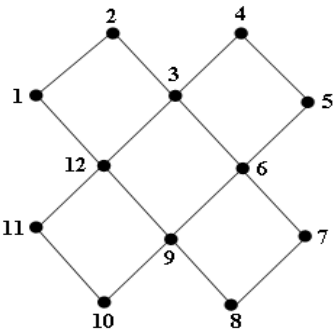

Figure. 1. *{ 1, 4, 7, 10 }, { 2, 5, 8, 11 }, { 3, 6, 9, 12 } are the distinct γ - sets.*

In [7], they have proved the following results,

R1. A graph G is JE if and only if,

i. γ (G) divides n.

ii. d (G) = n / γ (G), where d (G) denotes the domatic partition of G.

iii. G has exactly n / γ (G) distinct γ - sets.

In [8], the following result has been proved.

R2. A JE graph has no down vertex.

By using JE property, in [9], C. V. R. Harinarayanan et al., have defined strong JE graph and extended the results on JE to strong JE graphs. In [10], K. M. Dharmalingam et al., have studied about excellent – just excellent – degree equitable domination in graphs.

In [11] M. Yamuna and K. Karthika, have obtained the domatic number of the subdivision graph of a just excellent graph. They have proved the following results,

R3. Let G be a JE graph such that d (G) $\leq$ 3. Then G has no 2 – dominated vertex.

R4. There is no JE graph G such that d (G) = 2.

R5. If G is JE, then d ($G_{sd}uv$) = 2, if d (G) = 3 and d ($G_{sd}uv$) $\leq$ 3, if d (G) $\geq$ 4.

In [12], they defined a new graph called domatic subdivision stable. A graph G is said to be domatic subdivision stable (dss), if d (G) = d ($G_{sd}uv$), for all u, v $\in$ V (G), u $\perp$ v.

Example

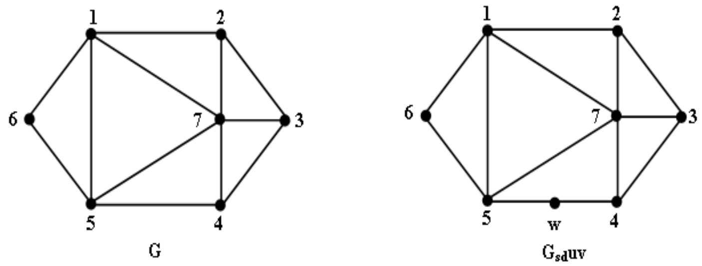

Figure. 2. The graph G is dss. Here d (G) = | { 5, 7 }, { 1, 4 }, { 2, 6, 3 } | = 3 and d (G_{sd} 54) = | { 5, 7 }, { 1, 4 }, { 2, 6, 3, w } | = 3. This is true for all u, v ∈ V(G), u ⊥ v.

In [12], they have proved the following results,

R6. Let G be any graph. Then d ($G_{sd}uv$) ≤ 3.

R7. If G is a graph such that d (G) ≥ 4, then G is not dss for every u, v ∈ V (G), u ⊥ v.

R8. When d (G) = 3, d ($G_{sd}uv$) = 3 if and only if there is a partition Z = { V_1, V_2, V_3 } for G such that

i. V_1 and V_2 are dominating sets for G such that u ∈ V_1, v ∈ V_2.
ii. v is 2 – dominated with respect to V_1.
iii. u is 2 – dominated with respect to V_2.
iv. V_3 dominates atleast G – { u } – { v }.

In 1991, Carrington et al. have defined unchanging edge addition (UEA), the graphs for which the domination number unchanged, when an arbitrary edge is added [3].

R9. A graph G ∈ UEA if and only if V^- is empty. Where V^- = { v ∈ V: γ (G – u) < γ (G) }.

II. RESULTS AND DISCUSSION

Theorem 1. If G is a JE graph such that d (G) = 3, then there is no partition Z = { V_1, V_2, V_3 } for G such that

i. V_1 and V_2 are dominating sets for G such that u ∈ V_1, v ∈ V_2.
ii. v is 2 – dominated with respect to V_1.
iii. u is 2 – dominated with respect to V_2.
iv. V_3 dominates atleast G – { u } – { v }.

Proof. If possible assume that there is a partition Z = { V_1, V_2, V_3 } for G such that the conditions of the theorem is satisfied.

Since G is JE,

a. G has no 2 – dominated vertex.
b. d(G) = n / γ (G), that is each V_i∈ d (G) is a γ - set.

Hence by conditions ii and iii, Z cannot be a domatic partition for G. Since the condition of the theorem is satisfied, we observe that V_1 and V_2 are dominating sets. Hence V_3 is not a dominating set for G. We have assumed that V_3 dominates atleast G – { u } – { v }. By our assumption,

i. v is 2 – dominated with respect to V_1, which implies V_1 dominates G $_{sd}$uv.
ii. u is 2 – dominated with respect to V_2, which implies V_2 dominates G $_{sd}$uv.
iii. V_3 ∪ { w } dominates G $_{sd}$uv.

That is d (G $_{sd}$uv) = | { V_1, V_2, V_3 ∪ { w } } | = 3, a contradiction, by R5. Hence there is no partition for G, which satisfies the conditions of the theorem.

Corollary. If G is JE, then G is not dss.

Proof. If d (G) ≥ 4, then by R7 , G is not dss.

If d (G) = 2, then G is not JE, by R4.
If d (G) = 3 and G is JE, then G has no 2 – dominated vertex by R3.
Also by Theorem 1, we know that there is no partition Z = { V_1, V_2, V_3 } for G, that satisfies the conditions of Theorem R8.
Hence if G is JE, then G is not dss.

Theorem 2. If G is JE, then d (G – u) = d (G) – 1, for all u $\in$ V (G).
Proof. We know that, in JE graph, every vertex is a level vertex, that is γ (G – u) = γ (G), for all u $\in$ V (G). Also d (G) = n / γ (G). | V (G – u) | = n – 1 and | γ (G – u) | = | γ (G) |. Hence n / γ (G) partition for G – { u } is not possible, which implies d (G – u) < d (G). Let d (G) = | { V_1 }, { V_2 }, …, { V_k } |. Let u $\in$ V_i, { V_1, V_2, …, V_{i-1}, V_{i+1}, …, $V_k \cup$ (V_i – { u }) } is a domatic partition for G – u, which implies d (G – u) = d (G) – 1.

Theorem 3. If G is a graph such that d (G) = k = | { V_1 }, { V_2 }, …, { V_k } |, k $\geq$ 3, then d ($G'_{sd}uv$) = 3, where G' = G + uv, for all u, v $\in$V (G), u $\in$ V_i, v $\in$ V_j, u is not adjacent to v, i$\neq$ j, i, j = 1, 2, …, k.
Proof. Let G be a graph such that d (G) = | { V_1 }, { V_2 }, …, { V_k } |. Let u $\in$ V_i, v $\in$ V_j. Let G′ = G + uv, u is not adjacent to v in G. So there is one vertex $u_1 \in V_i$, $u_1 \neq$ u such that $u_1 \perp$ v. Similarly there is one $v_1 \in V_j$, $v_1 \neq$ v such that $v_1 \perp$ v, which implies V_i and V_j are dominating set for $G'_{sd}uv$ also.
Let V_p = { $V_1 \cup V_2 \cup$, …, $\cup$ $V_{i-1} \cup V_{i+1}$, …, $\cup$ $V_{j-1} \cup V_{j+1} \cup$, …, V_k }. Vp $\cup$ { w } is a dominating set for $G'_{sd}uv$, since V_p is a dominating set for G′. { V_i, V_j, $V_p \cup$ { w } } is a domatic partition for $G'_{sd}uv$, which implies d ($G'_{sd}uv$) $\geq$ 3. Since P_3: uwv is a subgraph of $G'_{sd}uv$, by R6, we know that d ($G'_{sd}uv$)$\leq$ 3, which implies d ($G'_{sd}uv$) = 3.

Theorem 4. If G is a JE graph such that d (G) = k = | { V_1 }, { V_2 }, …, { V_k } |, k $\geq$ 3, then d (G') = k and d ($G'_{sd}uv$) = 3, where G' = G + uv, for all u, v $\in$V (G), u $\in$ V_i, v $\in V_j$, u is not adjacent to v, i$\neq$ j, i, j = 1, 2, …, k.
Proof. Let G be a JE graph and let d (G) = | { V_1 }, { V_2 }, …, { V_k } |. Since G is JE, we know that, G has no down vertex .
Since G has no down vertex, then by Theorem R9, γ (G + uv) = γ (G), for all u, v $\in$V (G), u is not adjacent to v.
By Theorem 3, we know that d ($G'_{sd}uv$) = 3.

Theorem 5. If G is a JE graph such that d (G) = 3, then d ($G'_{sd}uv$) = 2, for all u, v $\in$V (G), u, v $\in$ V_i, where G' = G + uv, i = 1, 2, 3, u is not adjacent to v.
Proof. Let G be a JE graph and d (G) = | { V_1 }, { V_2 }, { V_3 } |. Let u, v $\in V_i$, u is not adjacent to v. We know that d (G') = 3, by Theorem 4.
If possible assume that let d ($G'_{sd}uv$) = 3 = | { V_1' }, { V_2' }, { V_3' } |, where u $\in$ V_1', v $\in$ V_2', w $\in$ V_3'.
Claim. | V_1' |, | V_2' | > γ(G).
Proof. If possible assume that, | V_1' | = γ(G). $V_1' \neq$ V_i, since v $\in V_i$, v $\notin$ V_1'. V_1' is a γ - set for G also, which implies V_i and V_1' are two distinct γ - sets containing u, a contradiction as G is JE, which implies | V_1' |, | V_2' | > γ (G).
By claim |V_1' |, | V_2' | is atleast γ(G) + 1, which implies | V_1' | + | V_2' | + | V_3' | $\geq$ | V (G) | + 2, which is not possible, since | V ($G'_{sd}uv$) | = | V (G) | + 1. Hence d ($G'_{sd}uv$) $\neq$ 3, which implies d ($G'_{sd}uv$) = 2.

Theorem 6. If G is a JE graph such that d (G) = k = | { V_1 }, { V_2 }, …, { V_k } |, k $\geq$ 4,
then d ($G'_{sd}uv$) = 3, where G′ = G + uv, for all u, v $\in$V (G) and u, v $\in$ V_i, i = 1, 2, …,
k, u not adjacent to v.

Proof. Let G be a JE graph and let d (G) = | { V_1 }, { V_2 }, …, { V_k } |, k ≥ 4. Let u, v $\in V_i$, u not adjacent to v. Let G' = G + uv. Since G is JE, we know that d (G') = | { V_1 }, { V_2 }, …, { V_k }, by Theorem 4. Let $V_p = V_i - \{ v \} \cup V_x$, x ≠i, j, $V_q = V_j \cup \{ v \}$ and $V_r = V_1 \cup V_2 \cup, \ldots, \cup V_{i-1} \cup V_{i+1}, \ldots, \cup V_{j-1} \cup V_{j+1}, \ldots, V_{x-1} \cup V_{x+1}, \ldots, \cup V_k$. Since u is adjacent to v in G', $V_i - \{ v \}$ dominates v. Since V_x is a γ - set for G', there is one $u_1 \in V_x$, $u_1 \neq u$ such that u_1 dominates v. Hence v is 2 – dominated with respect to V_p, which implies V_p dominates $G'_{sd}uv$ also. Similarly since v is adjacent to u and V_j is a γ - set for G', u is 2 – dominated with respect to V_q, which implies V_q dominates $G'_{sd}uv$ also. Let $V_s = V_r \cup \{ w \}$. Since V_r is a dominating set for G' and V_s dominates $G'_{sd}uv$ also. Therefore d ($G'_{sd}uv$) = | { V_p }, { V_q }, { V_s } | = 3, by R6.

III. CONCLUSION

By Theorem 5 and 6, we conclude that

If G is JE such that d (G) = k, then d (G') = k, for all u, v $\in$ V (G), u is not adjacent to v, where G' = G + uv.

$$d (G'_{sd}uv) = \begin{cases} 2 & if\ d\ (\ G\ ) = 3 \\ 3 & if\ d\ (\ G\ ) \geq 4. \end{cases}$$

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